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## Impact of primary user duty cycle in generalized fading channels on spectrum sensing in cognitive radio

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## Abstract

Spectrum sensing is the key technology in cognitive radio for the efficient use of radio spectrum. The implementation of energy detection based spectrum sensing is done with the assumption of constant activity of the primary user. Practically, often the scenario occurs when primary user transmits signal in the frequency band of interest only for a fraction of the observation period. The paper aims at analyzing the degradation in performance of energy detection based sensors in the case of generalized  $\kappa$ - $\mu$  and  $\alpha$ - $\mu$  fading channels. The theoretical results indicate low reliable performance in the case of fading channel due to primary user duty cycle variation at low signal-to-noise ratios.

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**Keywords:** Cognitive radio; Spectrum sensing; Energy detector;  $\kappa$ - $\mu$  fading;  $\alpha$ - $\mu$  fading

## 1. Introduction

Cognitive radio (CR) is one of the recent trends in wireless communication, achieving a lot of attention among the researchers as well as industries. It can be viewed as a potential solution to the underutilization of licensed spectrum by primary user (PU) and enables the unlicensed/ secondary user (SU) to exploit the *white spaces* or *spectral holes* without affecting the quality-of-service (QoS) of the PU. Spectrum sensing is the key technology in CR, which makes it possible to opportunistically access the white spaces<sup>1</sup>. Several methods have been proposed in the literature<sup>2</sup>, such as energy detector, cyclo-stationary feature detector, matched filter detector, wavelet based detector, etc. for spectrum sensing.

Out of all the above mentioned techniques, energy detector outperforms all, by virtue of its low computational complexity and implementation cost. It does not require any prior knowledge about the format of the signal being transmitted by the PU. However, the main drawback of the energy detector technique lies in the fact that, the detection threshold is highly susceptible to the uncertainty of the background noise. Several assumptions are also imposed on the functionality of the energy detector. One such assumption, often found in the literature, is the constant activity state of the PU during the sensing interval<sup>3</sup>. Such scenarios occur when the PU signal may be non-stationary and change

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its state during the sensing interval. This leads to the significant performance degradation of the spectrum sensor. The impact of PU duty cycle in non-fading channel (AWGN channel) has been studied recently<sup>3</sup>. The performance evaluation of the spectrum sensing in the fading environment due to duty cycle variation of the primary user is not available in the literature.

Recently, some generalized fading distributions such as  $\kappa$ - $\mu$  and  $\alpha$ - $\mu$  have been proposed<sup>4,5</sup>, which are very useful models that provide a better fit to the experimental measured channel data. In the present work, assuming the  $\kappa$ - $\mu$  and  $\alpha$ - $\mu$  fading scenarios, the effect of PU duty cycle on the performance of the energy detection based spectrum sensing has been reported. Specifically, the receiver operating characteristic (ROC) has been obtained for different values of the duty cycle and a comparison of the performance degradation in spectrum sensing with the variation in duty cycle under both fading and non-fading scenarios is made.

## 2. System Model

In CR, the secondary user senses the PU signal in the band of interest and then tries to opportunistically access the unused spectrum. The signal received at the sensor node is a faded and noisy version of the original PU signal. Hence, the conventional spectrum sensing problem can be recognized as a binary hypothesis test as<sup>3</sup>:

$$y(n) = \begin{cases} u(n) & , H_0 \\ s(n) + u(n) & , H_1 \end{cases} \quad (1)$$

where,  $s(n) = h(n).x(n)$  is the faded version of the transmitted signal  $x(n)$ ,  $h(n)$  is the channel fading coefficient and  $u(n)$  is a zero-mean additive white Gaussian noise (AWGN) with a variance  $\sigma_u^2$ .

For the study of the impact of duty cycle on spectrum sensing performance, a new variable  $D$  is introduced<sup>3</sup>, which corresponds to the proportion of the observed signal in which PU signal is present,  $0 \leq D \leq 1$ ; where,  $D = 0$  is equivalent to the complete absence of PU signal and  $D = 1$  corresponds to the presence of PU signal for the entire sensing interval.

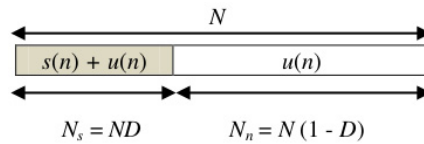


Fig. 1. Received signal with  $D < 1$ .

Fig. 1 illustrates the scenario, where PU signal is present partially during the sensing interval of length  $N$ . Here,  $N_s$  represents the portion containing the faded signal corrupted with noise, and  $N_n$  corresponds to the portion of the sensing interval in which PU was completely absent and hence it contains only the noise. This can be modeled as<sup>3</sup>:

$$y_D(n) = \begin{cases} s(n) + u(n) & , 0 \leq n \leq N_s \\ u(n) & , N_s + 1 \leq n \leq N \end{cases} \quad (2)$$

## 3. Energy Detection

In the present study, we have adopted the energy detection based spectrum sensing, due to its low computational complexity and lesser implementation cost. Fig. 2 illustrates the basic algorithm for energy detection based spectrum sensing. The decision variable  $T$  is the sum of the squares of sample amplitudes, in any particular sensing event, and is given as<sup>3</sup>:

$$T_i(y_i) = \sum_{n=1}^N |y_i(n)|^2 \quad (3)$$

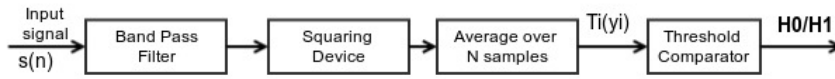


Fig. 2. Block diagram for energy detection based spectrum sensing.

The sensor uses this decision variable to map to the binary space  $H_0$  or  $H_1$ . In order to analyze the performance of the sensing scheme, the probability of false alarm,  $P_{fa}$ , and the probability of detection,  $P_d$ , are defined as follows<sup>3</sup>:

$$\begin{aligned} P_{fa} &= \text{Prob}\{H_1|H_0\} = \text{Prob}\{T_i(y_i) > \lambda|H_0\} \\ P_d &= \text{Prob}\{H_1|H_1\} = \text{Prob}\{T_i(y_i) > \lambda|H_1\} \end{aligned} \quad (4)$$

where,  $\lambda$  is the decision threshold. Without any loss of generality, assuming the distribution of the received signal  $y(n)$  to be Gaussian and for the condition of large number of samples  $N \gg 1$ , the decision variable may be modeled as<sup>3</sup>:

$$T_i(y_i) \sim \begin{cases} \mathcal{N}(N\sigma_u^2, 2N\sigma_u^4) & , H_0 \\ \mathcal{N}(N\sigma_u^2(1+\gamma), 2N\sigma_u^4(1+\gamma)^2) & , H_1 \end{cases} \quad (5)$$

where,  $\mathcal{N}(a, b)$  representing a Gaussian distribution with mean  $a$  and variance  $b$ , and  $\gamma$  denotes the signal-to-noise ratio (SNR), i.e.,  $\gamma = \frac{\sigma_s^2}{\sigma_u^2}$ ,  $\sigma_s^2$  is the variance of the signal  $s(n)$ . Utilizing (4) and (5), the probability of false alarm and the probability of detection may be written as<sup>3</sup>:

$$\begin{aligned} P_{fa} &= Q\left(\frac{\lambda - N\sigma_u^2}{\sqrt{2N\sigma_u^4}}\right) \\ P_d &= Q\left(\frac{\lambda - N\sigma_u^2(1+\gamma)}{\sqrt{2N\sigma_u^4(1+\gamma)^2}}\right) \end{aligned} \quad (6)$$

where,  $Q(\cdot)$  is the tail probability of the normal distribution. For a target false alarm probability,  $P_{fa,target}$ , the threshold can be computed as:

$$\lambda = (Q^{-1}(P_{fa,target})\sqrt{2N} + N)\sigma_u^2 \quad (7)$$

From Fig. 1, the portion of the sensing interval containing PU signal may be assumed to be normally distributed with zero-mean and variance equal to  $\sigma_{ON}^2 = \sigma_u^2(\gamma + 1)$ . The remaining portion is only consisting of noise with mean zero and variance equal to  $\sigma_u^2$ . The complete observed signal remains Gaussian distributed with zero mean and variance given as<sup>3</sup>:

$$\sigma_D^2 = \sigma_{ON}^2 D + \sigma_u^2(1 - D) = \sigma_u^2(\gamma D + 1) \quad (8)$$

Following the same analogy as (5), we may define the distribution of the decision variable under hypothesis  $H_1$  with duty cycle constraint,  $T_D(y_i)$  as<sup>3</sup>:

$$\begin{aligned} T_D &\sim \mathcal{N}(N\sigma_D^2, 2N\sigma_D^4) \\ \text{i.e., } T_D &\sim \mathcal{N}(N\sigma_u^2(\gamma D + 1), 2N\sigma_u^4(\gamma D + 1)^2) \end{aligned} \quad (9)$$

Hence, the probability of detection  $P_{DD}$ , the under duty cycle constraint, can be defined as<sup>3</sup>:

$$\begin{aligned}
 P_{DD} &= Q\left(\frac{\lambda - N\sigma_D^2}{\sqrt{2N\sigma_D^4}}\right) \\
 &= Q\left(\frac{Q^{-1}(P_{fa,target})}{\gamma D + 1} - \sqrt{\frac{N}{2}} \frac{\gamma D}{\gamma D + 1}\right) \\
 &= Q\left(Q^{-1}(P_{fa,target}) - \sqrt{\frac{N}{2}} \gamma D\right); \gamma \ll 1
 \end{aligned} \tag{10}$$

### 3.1. Energy detection over generalized fading channels

In case of fading channel, where the channel coefficient  $h(n)$  varies, the equation (10) gives the conditional probability of detection for an instantaneous signal-to-noise ratio  $\gamma$ . To find the unconditional detection probability,  $P_{DD}$  should be averaged over the probability density function of SNR  $f_{\Gamma}(\gamma)$ <sup>6</sup>:

$$P_{DD,fading} = \int_0^{\infty} P_{DD}(\gamma) f_{\Gamma}(\gamma) d\gamma \tag{11}$$

Here,  $P_{DD,fading}$  represents the probability of detection over fading channel, with duty cycle constraint. Various statistical distributions have been suggested in the literature, to model the short-term variation of the wireless channels. Few widely known distributions include Rayleigh, Rician (Nakagami- $n$ ), Nakagami- $m$ , Hoyt (Nakagami- $q$ ), Weibull, etc.<sup>7</sup>. Experimental results confirm that none of the above distributions gives a good fit for the practical scenarios<sup>4</sup>. Specially, the fit is very poor in the tail region of the distributions. In the present analysis we consider the generalized  $\kappa$ - $\mu$  and  $\alpha$ - $\mu$  fading distributions, which are well representative of many of the practical fading distributions and provide a very good fit to the data collected from the field measurements<sup>4,5</sup>.

#### 3.1.1. $\kappa$ - $\mu$ fading channel

The  $\kappa$ - $\mu$  is a generalized fading distribution<sup>4</sup>, representing the small-scale variation of the radio channel under line-of-sight (LOS) conditions. In the suggested distribution,  $\mu(> 0)$  defines the multipath clustering and  $\kappa(> 0)$  is the ratio of total power in dominant component (i.e., LOS) to the total power in scatter components (i.e., NLOS)<sup>4</sup>.

The probability density function (pdf) of the SNR ( $\gamma$ ) for the  $\kappa$ - $\mu$  distribution can be represented as<sup>6</sup>:

$$f_{\kappa-\mu}(\gamma) = \frac{\mu(1+\kappa)^{\frac{\mu+1}{2}}}{\kappa^{\frac{\mu-1}{2}} \exp[\kappa\mu] \sqrt{\gamma\bar{\gamma}}} \left(\frac{\gamma}{\bar{\gamma}}\right)^{\frac{\mu}{2}} \times \exp\left[-\mu(1+\kappa)\frac{\gamma}{\bar{\gamma}}\right] I_{\mu-1}\left[2\mu\sqrt{\kappa(1+\kappa)\frac{\gamma}{\bar{\gamma}}}\right] \tag{12}$$

where,  $I_{\nu}(\cdot)$  is the modified Bessel function of the first kind with order  $\nu$ ;  $\gamma$  and  $\bar{\gamma}$  represent the instantaneous SNR and the average SNR respectively. For certain values of  $\kappa$  and  $\mu$ , the given distribution can represent the pdf of the SNR of a number of well-known fading distributions. Table. 1 provides the values of  $\kappa$  and  $\mu$  for different channel models.

#### 3.1.2. $\alpha$ - $\mu$ fading channel

Similar to the  $\kappa$ - $\mu$  channel,  $\alpha$ - $\mu$  is also a generalized fading distribution<sup>5</sup>. Here,  $\alpha(> 0)$  represents the non-linearity of the propagation medium, which results from the non-homogeneous diffuse scatterers, and the parameter  $\mu(> 0)$  is related to the number of multipath clusters.

The pdf of the SNR for the  $\alpha$ - $\mu$  channel is given by<sup>5</sup>:

$$f_{\alpha-\mu}(\gamma) = \frac{\alpha\mu^{\mu}}{2\Gamma(\mu)\bar{\gamma}^{\frac{\alpha\mu}{2}}} \gamma^{\frac{\alpha\mu}{2}-1} \exp^{-\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\frac{\alpha}{2}}} \tag{13}$$

where  $\Gamma(\cdot)$  is the Gamma function, the other parameters have their usual significance. Table. 2 provides the values of  $\alpha$  and  $\mu$  for different channel models.

Table 1. Values of  $\kappa$  and  $\mu$  for different distributions.

Type of Distribution	$\kappa$	$\mu$
One-sided Gaussian	$\rightarrow 0$	0.5
Rayleigh	$\rightarrow 0$	1
Rician	$K$	1
Nakagami- $m$	$\rightarrow 0$	$m$

Table 2. Values of  $\alpha$  and  $\mu$  for different distributions.

Type of Distribution	$\alpha$	$\mu$
Nakagami- $m$	2	$m$
Weibull	$K$	1
Gamma	1	$a$
One-sided Gaussian	2	0.5
Rayleigh	2	1
Exponential	1	1

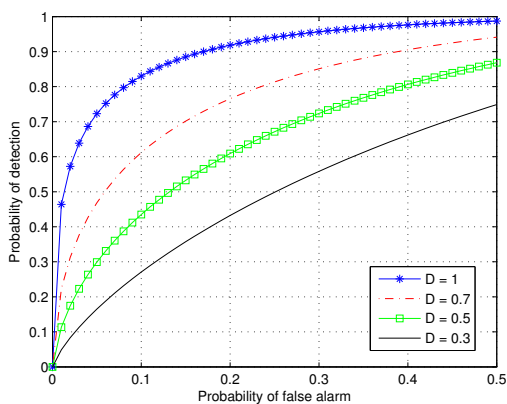
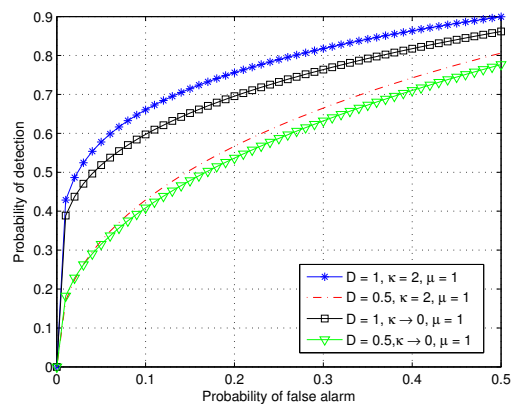
#### 4. Results and Discussion

The performance degradation of the energy detector in AWGN and generalized fading channels with duty cycle constraint is presented in this section. The performance has been evaluated in terms of degradation in the probability of detection,  $P_{DD}$ , as a function of the probability of false alarm,  $P_{fa}$ , as well as a variation of SNR.

In Fig. 3, the receiver operating characteristics (ROC) for the AWGN channel is shown with different values of the duty cycle. It is evident from the results that there is significant degradation in the performance when duty cycle  $D < 1$ . Fig. 4 depicts the ROC for various  $\kappa$ - $\mu$  channels with: i)  $\kappa \rightarrow 0, \mu = 1$  (Rayleigh), and ii)  $\kappa = 2, \mu = 1$  (Rice); with  $D = 1$  and  $D = 0.5$ . Similar results have been generated in Fig. 5 for the  $\alpha$ - $\mu$  channel with: i)  $\alpha = 2, \mu = 1$  (Rayleigh), and ii)  $\alpha = 2, \mu = 3$ ; with  $D = 1$  and 0.5. For all the cases, the average SNR,  $\bar{\gamma}$  has been considered as -10 dB and the number of samples in one sensing event,  $N = 1000$ .

Fig. 6 shows the variation in  $P_{DD}$  with  $\bar{\gamma}$  in  $\kappa$ - $\mu$  channel for  $\kappa \rightarrow 0, \mu = 1$ . The target false alarm probability is kept fixed at 0.01. The performance has been evaluated for two different values of duty cycle,  $D = 1$  and  $D = 0.5$ . The result indicates a significant degradation in  $P_{DD}$  when SNR varies from -15 dB to 10 dB with  $D = 0.5$  as compared to  $D = 1$ . In order to achieve the same performance as with  $N = 100$  and  $D = 1$ , the number of samples required for  $D = 0.5$  is 400. Similar is the case for the desired performance with  $N = 1000$  and  $D = 1$ , where the corresponding required increased value is  $N = 4000$ , for  $D = 0.5$ . Increasing the number of samples in each sensing event, results in higher sensing time and therefore the throughput of the secondary user network decreases.

Table. 3 shows a comparison of the performance degradation of the energy detector, in AWGN and fading channels. The table indicates the probability of detection with different values of duty cycle at  $\bar{\gamma} = -10$  dB SNR and  $N = 1000$ . It is evident from the table that the effect of duty cycle is more severe in the case of non-fading (AWGN) channel as compared to the fading channels. Moreover, in both the cases, we witness significant performance deterioration for lower value of the duty cycle.

Fig. 3. ROC in AWGN channel at  $\bar{\gamma} = -10$  dB and  $N = 1000$ .Fig. 4. ROC in  $\kappa$ - $\mu$  channel at  $\bar{\gamma} = -10$  dB and  $N = 1000$ .

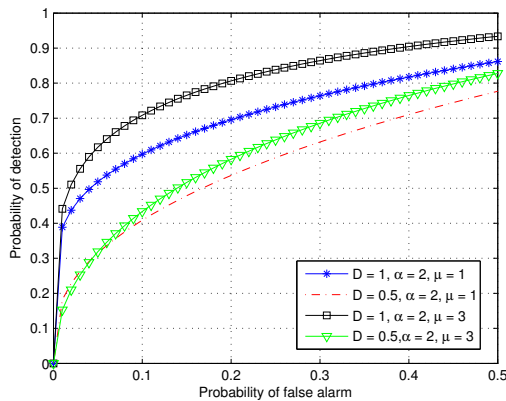


Fig. 5. ROC in  $\alpha$ - $\mu$  channel at  $\bar{\gamma} = -10$  dB and number of samples per sensing event  $N = 1000$ .

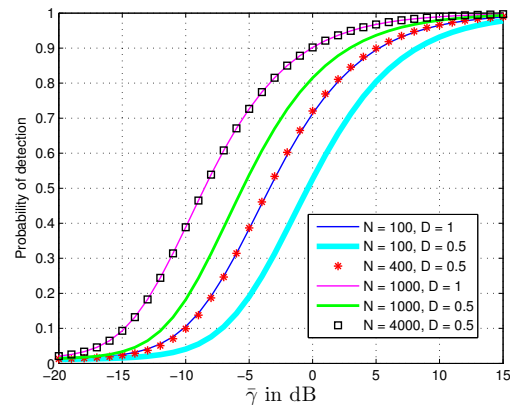


Fig. 6. Variation of  $P_{DD}$  with SNR for  $P_{fa,target} = 0.01$  in  $\kappa$ - $\mu$  channel with  $\kappa \rightarrow 0$  and  $\mu = 1$  (Rayleigh case).

Table 3. Probability of detection in AWGN and fading channel for different duty cycle (SNR = -10 dB and  $N = 1000$ ).

Duty Cycle ( $D$ )	$P_{DD}$ for $P_{fa,target} = 0.01$	
	AWGN Channel	$\kappa$ - $\mu$ / $\alpha$ - $\mu$ channel $\kappa \rightarrow 0, \mu = 1$ $\alpha = 2, \mu = 1$ , (Rayleigh case)
1	0.46403	0.38872
0.8	0.29546	0.31618
0.6	0.16238	0.22958
0.4	0.076083	0.1329
0.2	0.030113	0.045953

## 5. Conclusion

In this paper, we have analyzed the impact of primary user duty cycle variation on the performance of energy detection based spectrum sensing in generalized fading channels. Energy detector performs poorly, while detecting a non-stationary primary signal occupying only a fraction of the sensing period. For a given probability of false alarm, the degradation in detection probability is more in case of non-fading channel as compared to the fading channel. Since, in practical scenarios, it is difficult for the secondary user to gain knowledge of the primary user activity pattern, the detection performance calculated by the conventional detectors will not reflect the actual performance while sensing a non-stationary primary user. This results in unreliable system operation, when the secondary networks are designed to achieve the minimum detection requirements.

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